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PRACTICE 1 – IMPACTS

VIBRATIONS

– BMEGEMMBXM4 –

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Example

In Fig. 1, an 1 DoF oscillator is shown that consists of a beam of mass m_2 and a torsional spring of stiffness k_t. The beam can rotate about the pin at point A. The system is in the gravitational field, the preload of the torsional spring ensures that the horizontal position of the beam is the equilibrium of the oscillator. The vibration of the steady beam is induced by the impact between the beam and the lumped mass m_1. Before the impact, the lumped mass free falls from the height h starting with zero speed.

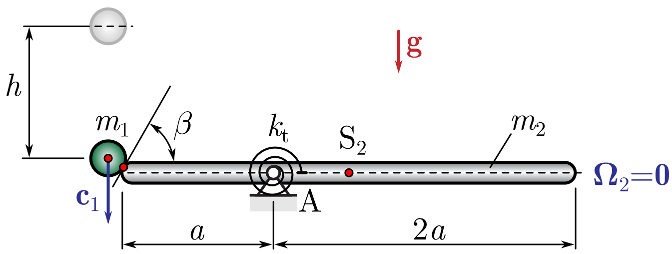

Fig. 1: The investigated system

Data

$m_1 = 6\,\mathrm{kg}$	$m_2 = 6\,\mathrm{kg}$
$a = 0.3\,\mathrm{m}$	$\beta = 60\,^\circ$
$h = 0.115\,\mathrm{m}$	$e = 1$

Task

1. Determine the velocity of the lumped mass and the angular velocity of the beam after the impact!

Made by Dénes Takács

Solution

Task 1

First, the velocity of the lumped mass has to be determined in the time moment before the impact. Based on the work-energy theorem:

$$T_1 - T_0 = W_{01}\,, \tag{1}$$

where T_0 and T_1 are the kinetic energies before and after the free fall, respectively. Since, before the free fall, the speed of the lumped mass is zero, the kinetic energy $T_0 = 0$. The mechanical work W_{01} between the two time instants consist of the work of the gravity. Namely:

$$\frac{1}{2}m_1 c_1^2 = m_1 g h \quad \Rightarrow \quad c_1 = \sqrt{2gh} = 1.5\,\mathrm{m/s}\,. \tag{2}$$

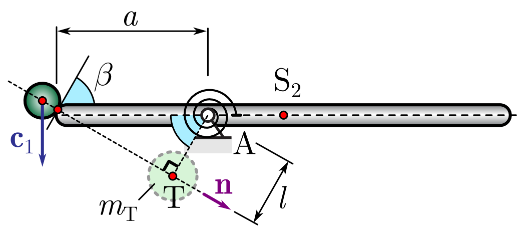

Fig. 2: Reduction of the impact

The normal line of the impact is perpendicular to the common tangent of the connecting surfaces of the lumped mass and the beam, see Fig. 2. Since the beam has a fix point at A, we have to calculate its reduced mass in order to handle the problem as centric impact. The center point T of the impact can be determined by drawing a perpendicular to the normal line from the fix point A. Based on Fig. 2, the distance between the fix point and the center of impact:

$$l = \overline{\mathrm{AT}} = a\cos\beta = 0.15\,\mathrm{m}\,, \tag{3}$$

and the reduced mass is

$$m_\mathrm{T} = \frac{\theta_A}{l^2} = \frac{\frac{1}{12}m_2(3a)^2 + m_2(\frac{a}{2})^2}{a^2\cos^2\beta} = 24\,\mathrm{kg}\,, \tag{4}$$

where θ_A is the mass moment of inertia of the beam with respect to the rotational axis at point A.

The velocity of the center point T before the impact:

$$c_{\mathrm{Tn}} = l\Omega_2 = 0 \tag{5}$$

since the beam is steady. Based on Fig. 3, the velocity of the lumped mass can be decomposed as

$$\begin{aligned} c_{1\mathrm{n}} &= c_1\cos\beta = 0.75\,\mathrm{m/s}\,,\\ c_{1\mathrm{t}} &= c_1\sin\beta = 1.3\,\mathrm{m/s}\,. \end{aligned} \tag{6}$$

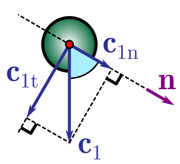

Fig. 3: Decomposition of the velocity of the lumped mass

Now, the problem can be managed as a simple centric impact, in which the masses m_1 and m_T have an impact with the velocities $c_{1\mathrm{n}}$ and c_{Tn}. This simple problem can be solved easily by using the Maxwell diagram.

Usually, the Maxwell diagram is used as a guide (we do not draw it with precise scales) to obtain the formulas by which the velocities after the impact can be determined. Thus, we calculate the velocity of the common center of gravity of the two bodies, which does not change during the impact due to the conservation of the linear momentum:

$$c_{\mathrm{Sn}} = v_{\mathrm{Sn}} = \frac{m_1 c_{1\mathrm{n}} + m_{\mathrm{T}} c_{\mathrm{Tn}}}{m_1 + m_{\mathrm{T}}} = 0.15\,\mathrm{m/s}\,. \tag{7}$$

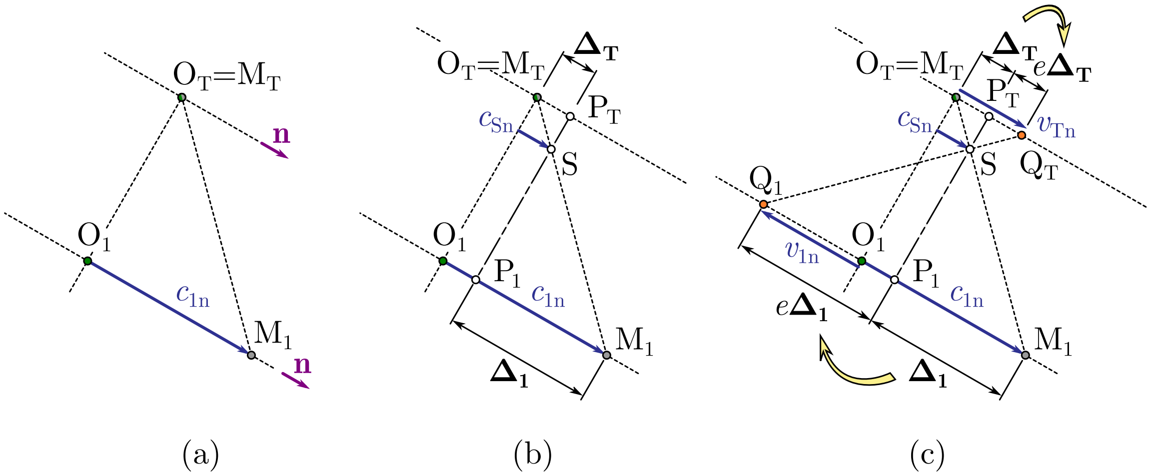

Fig. 4: The construction of the Maxwell diagram

The construction of the simplified Maxwell diagram is shown in Fig. 4. Two lines parallel with the normal directions are drawn, and the normal velocities of the masses are measured to them from the origins $\mathrm{O_1}$ and $\mathrm{O_T}$. This determines the points $\mathrm{M_1}$ and $\mathrm{M_T}$, see panel (a). As a second step, the common center of gravity S is marked based on the ratio of the masses: $\overline{\mathrm{SM_T}}/\overline{\mathrm{SM_1}} = m_1/m_{\mathrm{T}}$. The distances $\Delta_1 = \overline{\mathrm{M_1P_1}}$ and $\Delta_{\mathrm{T}} = \overline{\mathrm{M_TP_T}}$ (see panel (b)) are related to the speed changes when both masses reach the speed of the common center of gravity, i.e., when the local deformations caused by the impact are maximal. The coefficient of restitution e defines how the deformations turn back to speed, namely $e\Delta_1$ and $e\Delta_{\mathrm{T}}$ are measured to the opposite side of the $\mathrm{P_1P_T}$ line, see panel (c). The normal velocities after the impact are related to the distances $\overline{\mathrm{O_1Q_1}}$ and $\overline{\mathrm{O_TQ_T}}$.

Thus, the normal components of the velocities after the impact can be obtained with the help of the diagram:

$$\begin{aligned} v_{1\mathrm{n}} &= c_{\mathrm{Sn}} - e\Delta_1 = c_{\mathrm{Sn}} - e(c_{1\mathrm{n}} - c_{\mathrm{Sn}}) = -0.45\,\mathrm{m/s}\,,\\ v_{\mathrm{Tn}} &= c_{\mathrm{Sn}} + e\Delta_{\mathrm{T}} = (1+e)c_{\mathrm{Sn}} = 2c_{\mathrm{Sn}} = 0.3\,\mathrm{m/s}\,. \end{aligned} \tag{8}$$

Let us turn back to the original problem. The velocity of the lumped mass after impact:

$$\boxed{v_{1\mathrm{n}} = -0.45\,\mathrm{m/s} \quad \text{and} \quad v_{1\mathrm{t}} = c_{1\mathrm{t}} = 1.3\,\mathrm{m/s}\,.} \tag{9}$$

The velocity of the center point T is calculated, from which the angular velocity of the beam after the impact:

$$\boxed{\omega_2 = \frac{v_{\mathrm{Tn}}}{l} = 2\,\mathrm{rad/s} \quad (\circlearrowleft)\,.} \tag{10}$$